Workshop on Generic Structures

IM CAS, Prague

24-30 October 2021

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About

A mathematical structure could be called generic if it can be built by an infinite process which is as complicated as possible. Such structures were first identified in model theory (Fraïssé limits), nevertheless, currently the most general framework is metric-enriched category theory. This framework allows for exploring generic structures appearing in topology and functional analysis, including C*-algebra theory. Perhaps one of the most important attributes of generic structures is their uniqueness, up to isomorphism. This leads to sometimes unexpected results showing that two arbitrary structures, sharing a certain mild extension property, are actually the same.

The workshop is devoted to recent results and current problems around the theory of generic structures. The aim is to gather specialists working in the area as well as students and young researchers interested in the topic.

The event is taking place within the framework of the EXPRO project led by Wiesław Kubiś: Abstract Convergence Schemes And Their Complexities (Czech Science Foundation, 20-31529X).
Webpage: `http://expro.math.cas.cz`.

Organizing committee

Adam Bartoš, Tristan Bice, Wiesław Kubiś, Christian Pech and Beata Kubiś (project manager).

Location

Institute of Mathematics of the Czech Academy of Sciences, Žitná 25, 115 67 Praha 1, Czechia.

All talks will be held in the blue lecture room, rear building, ground floor.

See below for a map including some local eateries (note (1) is vegetarian, (2)-(5) serve typical Czech food, (6)-(8) offer Asian cuisine and (9)-(10) are also good but a bit on the expensive side).

IMPORTANT NOTE: Thursday October 28 is a Czech holiday (Independence Day) so some restaurants may not be open or have their usual lunch menu.

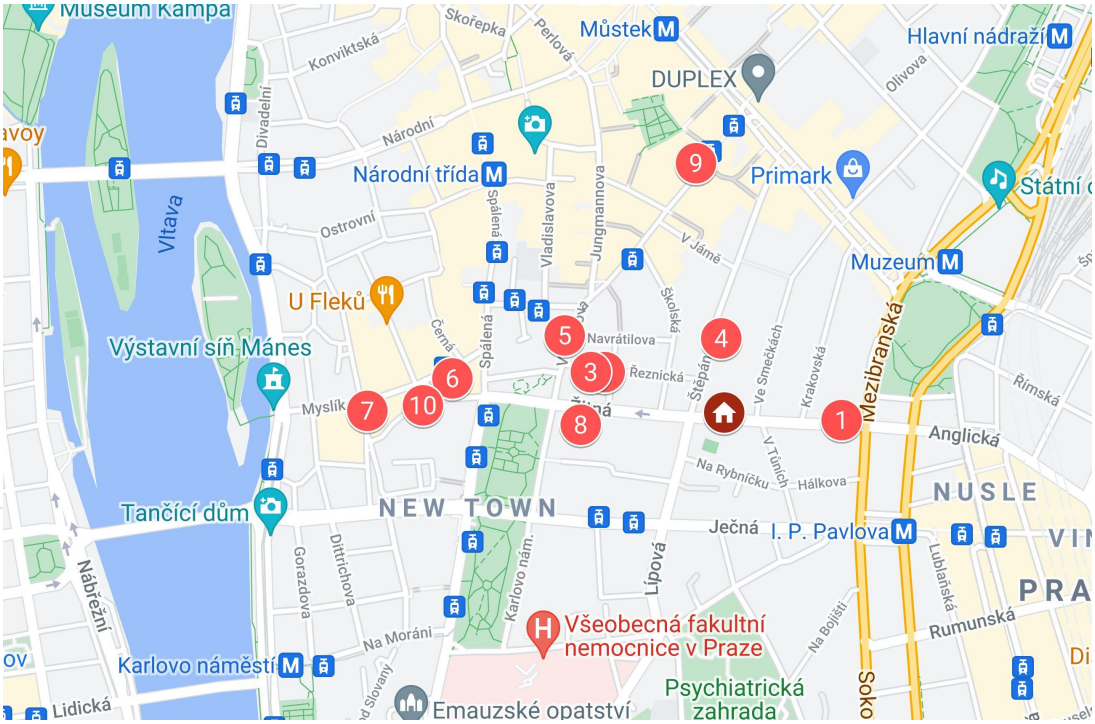
Prague – restaurants

Lunch restaurants

1. Palo Verde Bistro
2. Příčný řez
3. Jiná krajina
4. Steak restaurant Titanic
5. Cvikovský pivovar
6. The Nest
7. Lemon Leaf
8. DuHa Sushi Žitná
9. Styl&Interier
10. QQ Asian Kitchen

Home

Institute of Mathematics of the Czech Academy of Sciences



List of Participants

Speakers

Vadim Alekseev	TU Dresden
Marek Cúth	Charles University / IM CAS
Michal Doucha	IM CAS
Mirna Džamonja	IRIF, University of Paris
Saeed Ghasemi	York University / IM CAS
Jan Hubička	Charles University
Bhishan Jacelon	IM CAS
Jamal Kawach	Charles University
Wiesław Kubiś	IM CAS
Franz-Viktor Kuhlmann	University of Szczecin
Boriša Kuzeljević	University of Novi Sad
Aleksandra Kwiatkowska	University of Münster / University of Wrocław
Maciej Malicki	Institute of Mathematics, Polish Academy of Sciences
Dragan Mašulović	University of Novi Sad
Maja Pech	University of Novi Sad
Noé de Rancourt	Charles University
Marcin Sabok	McGill University
Paul Szeptycki	York University
Stevo Todorčević	University of Toronto / Institut de mathématiques de Jussieu / Mathematical Institute, SASA

Attendees (some online only)

M. Ali Asadi-Vasfi	IM CAS
Antonio Avilés López	University of Murcia
Adam Bartoš	IM CAS
Dana Bartošová	University of Florida
Tristan Bice	IM CAS
Jan Bíma	Charles University
Sam Braunfeld	Charles University
David Chodounský	IM CAS
Keegan Dasilva Barbosa	University of Toronto
Ivan Di Liberti	IM CAS
Martin Doležal	IM CAS
Jan Dudák	Charles University
Marián Fabian	IM CAS
Mohammad Golshani	IPM, Tehran
David Hartman	Charles University
Rouholah Hoseini Nave	Shahid Bahonar University of Kerman
Ziemowit Kostana	University of Warsaw
Jan Krajíček	Charles University
Andrey Krutov	IM CAS
Andrzej Kucharski	University of Silesia
Katarzyna Kuhlmann	University of Szczecin
Ondřej Kurka	IM CAS
Chris Lambie-Hanson	IM CAS
Natalia Maślany	Jagiellonian University
Vladimír Müller	IM CAS
Piotr Nowakowski	University of Łódź
Bojana Pantić	University of Novi Sad
Jean Kyung Park	University of Florida
Christian Pech	IM CAS
Veronika Pitrová	Jan Evangelista Purkyně University
Paulina Radecka	IM CAS
Jakub Rondoš	Charles University
Tommaso Russo	IM CAS
Jiří Spurný	Charles University
Karen Strung	IM CAS
Jarosław Swaczyna	Łódź University of Technology
Sławomir Turek	UKSW
Benjamin Vejnar	Charles University
Lauren Wickman	University of Florida
Kentaro Yamamoto	ICS CAS

Timetable

Monday, October 25

9:45–10:30	**Wiesław Kubiś** IM CAS	Abstract evolution systems as a convenient framework for generic structures
10:30–11:15	**Coffee**	
11:15–12:00	**Jan Hubička** Charles University	Big Ramsey degrees of homogeneous structures
12:00–13:45	**Lunch**	
13:45–14:15	**Coffee**	
14:15–15:00	**Dragan Mašulović** University of Novi Sad	On big (dual) Ramsey degrees in free algebras and cofree coalgebras
15:15-17:15	**Discussion Session: The various faces of Fraïssé theory**	

Tuesday, October 26

9:00–9:45	**Maciej Malicki** Institute of Mathematics, Polish Academy of Sciences	Large conjugacy classes and weak amalgamation
9:45–10:30	**Aleksandra Kwiatkowska** University of Münster / University of Wrocław	Simplicity of the automorphism groups of ultrahomogeneous structures
10:30–11:15	**Coffee**	
11:15–12:00	**Marcin Sabok** McGill University	Perfect matchings in hyperfinite graphings
12:00–13:45	**Lunch**	
13:45–14:15	**Coffee**	
14:15–15:00	**Franz-Viktor Kuhlmann** University of Szczecin	About hyperfields
15:15-17:15	**Discussion Session: Generic structures in set theory and topological groups**	

Wednesday, October 27

9:00–9:45	**Mirna Džamonja** IRIF, University of Paris	Morass-generic structures
9:45–10:30	**Boriša Kuzeljević** University of Novi Sad	Tukey order of directed sets of cofinality ω_2
10:30–11:15	**Coffee**	
11:15–12:00	**Stevo Todorčević** University of Toronto / Institut de mathématiques de Jussieu / SASA	Forcing with Copies of the Rado and Henson Graphs
12:00–13:45	**Lunch**	
13:45–14:15	**Coffee**	
14:15–15:00	**Paul Szeptycki** York University	A topological space from a $\square(\kappa)$ sequence related to convergence and cardinal invariants of the G_δ topology
15:00–15:10	**Short Break**	
15:10–15:30	**Bojana Pantić** University of Novi Sad	A Fraïssé theorem in the light of polymorphism-homogeneity
18:00–22:00	**Informal Meeting**	

Thursday, October 28

9:00–9:45	**Noé de Rancourt** Charles University	Weak Fraïssé classes and $\aleph_0$-categoricity
9:45–10:30	**Marek Cúth** Charles University / IM CAS	Interplay between generic Banach spaces and Banach spaces with G_δ isometry class
10:30–11:15	**Coffee**	
11:15–12:00	**Jamal Kawach** Charles University	On the isometry group of Pełczyński's universal basis space
12:00–13:45	**Lunch**	
13:45–14:15	**Coffee**	
14:15–15:00	**Michal Doucha** IM CAS	Banach-Lie groups: an interesting class of Polish groups
15:15-17:15	**Discussion Session: Fraïssé limits in Banach spaces**	

Friday, October 29

9:00–9:45	**Vadim Alekseev** TU Dresden	Amenability, hyperfiniteness and (Fraïssé) limits
9:45–10:30	**Bhishan Jacelon** IM CAS	Generic classifiable C*-algebras
10:30–11:15	**Coffee**	
11:15–12:00	**Saeed Ghasemi** York University / IM CAS	The Jiang-Su algebra as a Fraïssé limit
12:00–13:45	**Lunch**	
13:45–14:15	**Coffee**	
14:15–15:00	**Maja Pech** University of Novi Sad	Classification methods for homomorphism-homogeneous structures
15:15-17:15	**Discussion Session: Generic structures in C*-algebra theory**	

List of Abstracts

Monday October 25

Abstract evolution systems as a convenient framework for generic structures

Wiesław Kubiś

IM CAS

An evolution system is a category with a distinguished object and a distinguished class of arrows, called transitions. This simple concept turns out to be a convenient model for generic limits. It also generalizes abstract rewriting systems, where the quality or cost of the rewriting process plays a significant role. We shall explain these ideas, showing some relevant examples.

Big Ramsey degrees of homogeneous structures

Jan Hubička

Charles University

We say that countably-infinite relational structure M has finite big Ramsey degrees if for every finite substructure A of M there exists finite integer $T_M(A)$ (the big Ramsey degree of A in M) such that for every finite colouring of copies of A in M there exists a copy M' of M in M such that the copies of A in M' have at most $T_M(A)$ many different colours.

It was shown by Laver, in late 1960's, that the big Ramsey degrees of the ordered rationals are finite. Subsequently precise degrees were computed by Devlin. This construction was adjusted to the Rado graph with precise degrees given by Laflamme-Sauer-Vuksanovic. The main tool used in these results is the Milliken Tree Thorem.

Recently the whole area was revitalized by results of Dobrinen (on big Ramsey degrees of Henson graphs) and Zucker (on free amalgamation classes of structures in binary languages) and by connections to topological dynamics. These results are based on structured variants of the Milliken Tree Theorem which is proved by methods inspired by Harrington's forcing proof of the Halpern-Lauchli theorem.

We discuss recent advancements in the area based on a new connection to the Carlson-Simpson theorem which makes it possible to analyze big Ramsey degrees of many additional structures (including partial orders, generalised metric spaces, random structures of higher arity and more). This is joint work with Martin Balko, David Chodounsky, Natasha Dobrinen, Matěj Konečný, Jaroslav Nešetřil, Stevo Todorčević, Lluis Vena and Andy Zucker.

On big (dual) Ramsey degrees in free algebras and cofree coalgebras

Dragan Mašulović

University of Novi Sad

In this talk we present a completely new set of strategies for proving that certain universal structures have finite big (dual) Ramsey degrees. Our strategies are derived from the fact that right adjoints preserve the Ramsey property, while left adjoints preserve the dual Ramsey property.

We consider classes of Eilenberg-Moore coalgebras for a comonad and, dually, classes of Eilenberg-Moore algebras for a monad. We then show that every finite coalgebra has finite big Ramsey degree in the cofree coalgebra on countably many free generators and, dually, that every finite algebra has finite dual big Ramsey degree with respect to Borel colorings in the free algebra on countably many free generators.

Tuesday October 26

Large conjugacy classes and weak amalgamation

Maciej Malicki

Institute of Mathematics, Polish Academy of Sciences

Ivanov, and later Kechris and Rosendal, characterized the existence of dense or comeager (diagonal) conjugacy classes in automorphism groups of certain classes of Fraïssé limits. I will show how these results can be extended to limits of weak Fraïssé classes, using an approach developed by Krawczyk and Kubiś. I will discuss homogenizability of weak Fraïssé classes, i.e., existence of a definable expansion to a Fraïssé class. I will also examine automorphism groups of ordered Fraïssé classes with the question in mind whether there exists a non-archimedean extremely amenable group with ample generics.

Simplicity of the automorphism groups of ultrahomogeneous structures

Aleksandra Kwiatkowska

University of Münster / University of Wrocław

We prove simplicity for the automorphism groups of order and tournament expansions of ultrahomogeneous structures like the bounded rational Urysohn metric space, the random graph, and the random poset. This is joint work with F. Calderoni and K. Tent.

Perfect matchings in hyperfinite graphings

Marcin Sabok

McGill University

We characterize hyperfinite bipartite graphings that admit measurable perfect matchings. In particular, we prove that every regular hyperfinite one-ended bipartite graphing admits a measurable perfect matching. We give several applications of this result. We extend the Lyons-Nazarov theorem by showing that a bipartite Cayley graph admits a factor of iid perfect matching if and only if the group is not isomorphic to the semidirect product of $\mathbb{Z}$ and a finite group of odd order, answering a question of Kechris and Marks in the bipartite case. We also answer a question of Bencs, Hrušková and Tóth arising in the study of balanced orientations in graphings. Finally, we show how our results generalize and lead to a simple approach to recent results on measurable circle squaring. Joint work with Matt Bowen and Gabor Kun

About hyperfields

Franz-Viktor Kuhlmann

University of Szczecin

Hyperfields were introduced in 1957 by Marc Krasner. They differ from fields in that their addition is multivalued, i.e., the sum of two elements is a subset of the hyperfield. Given a field K, a construction of a corresponding hyperfield presented by Krasner is to take a subgroup G of its multiplicative group $K^\times$, then adjoin an element 0 to the multiplicative group $K^\times/G$ and define on $K/G := K^\times/G \cup \{0\}$ a multivalued addition by setting, for any $a, b \in K$,

$$aG + bG := \{(c+d)G \mid c \in aG \text{ and } d \in bG\}.$$

Recently, hyperfields have attracted much attention in connection with algebraic geometry, tropical geometry, number theory, quadratic forms and even coding theory.

If K is a valued field and G is any subgroup of the units of the valuation ring, then the hyperfield inherits a function that is very similar to a valuation, and we then call it a valued hyperfield. These are particularly interesting for us as they arise in two different ways. 1) In the model theory of valued fields: several classes of valued fields admit quantifier elimination with respect to RV structures, as shown by J. Flenner. RV structures are essentially hyperfields, but with the multivalued addition of the latter encoded by a ternary relation that is more handy for model theoretic purposes. 2) M. Marshall and P. Gladki have studied ordered hyperfields. K. Kuhlmann in collaboration with our PhD students has shown that, as in the case of ordered fields, also in hyperfields natural valuations can be associated to orderings, and important properties carry over.

I will introduce valued hyperfields and discuss some results and open problems of the recent research on hyperfields.

This is ongoing joint work of myself and Katarzyna Kuhlmann with PhD students Hanna Stojałowska and Alessandro Linzi.

Wednesday October 27

Morass-generic structures

Mirna Džamonja

IRIF, University of Paris

We discuss joint work with Wiesław Kubiś on a specific way of constructing structures of size $\aleph_1$ using finite approximations, namely by organising the approximations along a simplified morass. We demonstrate a connection with Fraïssé limits and show that the naturally obtained structure of size $\aleph_1$ is homogeneous. Moreover, this is preserved under expansions, which leads us to a partial answer to a question of Bassi and Zucker. We give some examples of interesting structures constructed, such as an antimetric space of size $\aleph_1$. Finally, we comment on the situation when one Cohen real is added.

Tukey order of directed sets of cofinality ω_2

Boriša Kuzeljević

University of Novi Sad

We will first introduce the notion of Tukey reducibility between directed partially ordered sets. Then we will present some preliminary results about the Tukey order of the class of directed sets of cofinality ω_2. In particular, we will present the simplest types in this class and show which of them form a gap in this ordering. This is joint work with Stevo Todorčević.

Forcing with Copies of the Rado and Henson Graphs

Stevo Todorčević

University of Toronto / Institut de mathématiques de Jussieu / Mathematical Institute, SASA

We present joint works with Osvaldo Guzmán and Miloš Kurulić on special cases of the following general problem. An indivisible ultra homogeneous countable structure F has natural quotient algebra associated with, the quotient algebra of the power-set algebra of F modulo the ideal I_F of subsets of F that do not contain. isomorphic copies of F (the ideal of F-scattered sets). We analyze to which extent F determines $P(F)/I_F$ as a forcing notion. More precisely, if we let $B(F)$ denote the regular-open. algebra of $P(F)/I_F$, we analyze the functor $F \to B(F)$ from the class of countable ultra homogeneous indivisible structures into the class of complete Boolean algebras.

A topological space from a $\square(\kappa)$ sequence related to convergence and cardinal invariants of the G_δ topology

Paul Szeptycki

York University

A topological space defined from a square sequence is used to answer questions of Bella concerning cardinal invariants of G_δ modifications and, under some additional assumptions, other questions of Arhangel'skii concerning spaces with strong Fréchet-Urysohn properties.

A Fraïssé theorem in the light of polymorphism-homogeneity

Bojana Pantić

University of Novi Sad

The notion of homogeneity has established itself firmly over the years as one of the basic model theoretic properties. Initially, it rose to fame upon Fraïssé's construction of (homogenous) structures as "limits" of their ages. Development of a rich theory ensued. Alongside this, people began to take an interest in homogeneity itself. In recent years, the latter notion was generalised in various ways. One of them is so-called polymorphism-homogeneity. More specifically, a structure is called polymorphism-homogeneous if every local polymorphism can be extended to a global one. In this talk I will be reporting on a brand new Fraïssé-type theorem for polymorphism-homogenous relational structures. This is joint work with Maja Pech.

Thursday October 28

Weak Fraïssé classes and $\aleph_0$-categoricity

Noé de Rancourt

Charles University

Weak Fraïssé classes are a generalization of Fraïssé classes, and can be characterized as those classes admitting a generic countable object. This generic object acts as a Fraïssé limit, and a generalized Fraïssé correspondance can be proved in this setting. My talk will start with a short survey on the theory of weak Fraïssé classes, in a model-theoretic context. Then, I will present results linking this theory with the notion of $\aleph_0$-categoricity, namely: if a countable structure X is $\aleph_0$-categorical, then its age is a weak Fraïssé class, and the Fraïssé limit of its age is itself $\aleph_0$-categorical.

Interplay between generic Banach spaces and Banach spaces with G_δ isometry class

Marek Cúth

Charles University / IM CAS

In the first part I will talk about a recent project, where with M. Doucha, M. Doležal and O. Kurka we developed a new natural topological approach to coding of separable Banach spaces. It makes meaningful questions such as which Banach spaces are the easiest to define, up to isometry (and also up to isomorphism), or which classes of Banach spaces are the easiest to define in the descriptive set theoretic framework.

In the second part I will connect this research with the study of generic structures. This will be based on work in progress with M. Doucha and N. de Rancourt. The main result so far is that we are able to characterize Banach spaces with G_δ isometry class as exactly those Banach spaces which satisfy a certain “Fräissé-like” condition.

On the isometry group of Pełczyński’s universal basis space

Jamal Kawach

Charles University

We will outline "1-complemented" versions of the Fraïssé correspondence and the Kechris-Pestov-Todorčević correspondence for Banach spaces in which embeddings are replaced by sections (i.e. embeddings with 1-complemented range) or by embedding-projection pairs. In particular, this allows us to prove that the isometry group of a renorming of Pełczyński’s universal basis space is extremely amenable. To do this, we show that the class of finite-dimensional normed spaces is a 1-complemented Fraïssé class with the approximate Ramsey property. This is joint work in progress with Jordi López-Abad.

Banach-Lie groups: an interesting class of Polish groups

Michal Doucha

IM CAS

The most studied class of non-locally compact Polish groups in descriptive set theory, topological dynamics and related areas are the automorphism groups of various standard homogeneous structures. This includes groups such as S_∞, the unitary group of a separable infinite-dimensional Hilbert space, the isometry group of the Urysohn space, etc. Going somewhat against the main topic of the workshop, we will try to describe several very nice properties of Polish groups having a quite different origin - separable connected Banach-Lie groups. We show that they have a metric that is both maximal and minimal, in the sense of Rosendal, so they have well-defined large scale and small scale geometries. We also show that they are a natural source of examples of groups with the Haagerup property as well as properties (T) and (FH).

The talk will be based on joint work with Hiroshi Ando and Yasumichi Matsuzawa.

Friday October 29

Amenability, hyperfiniteness and (Fraïssé) limits

Vadim Alekseev

TU Dresden

In this talk, I'll give an overview of known results concerning amenability of various "limit structures" with particular focus on Fraïssé limits and discuss some ways that metric model theory could contribute further to understanding of operator algebras and structures from dynamical systems and ergodic theory.

Generic classifiable C*-algebras

Bhishan Jacelon

IM CAS

Using the continuous version of Fraïssé theory developed for metric structures, many C*-algebras of importance in the Elliott classification programme have recently been described as generic objects. In this talk, I will give a brief introduction to the Elliott programme, explain why its relationship with Fraïssé theory is a natural one, and provide examples of these Fraïssé limit C*-algebras. In particular, I will discuss my and Alessandro Vignati's constructions of the Razak-Jacelon and Santiago algebras (both of which play central roles in the classification of stably projectionless C*-algebras) as well as our hopes for potential applications to analysis of their internal structure.

The Jiang-Su algebra as a Fraïssé limit

Saeed Ghasemi

York University / IM CAS

In recent years some important C*-algebras have been flagged and studied as Fraïssé limits. Among these, the Jiang-Su algebra, usually denoted by $\mathcal{Z}$, stands out for its salient role in "Elliott's classification program" of separable nuclear C*-algebras by K-theoretic data. Perhaps the most important property of the Jiang-Su algebra is that it is "strongly self-absorbing". That is, it is isomorphic to its (minimal) tensor product with itself, in a "strong" sense. In their original paper from 1999, Jiang and Su introduced $\mathcal{Z}$ and already prove that $\mathcal{Z}$ is strongly self-absorbing. However, their proof uses heavy tools from classification theory, such as KK-theory and it is quite involved. I will present a self-contained and direct proof for the fact that $\mathcal{Z}$ is strongly self-absorbing, by establishing a general connection between the strongly self-absorbing C*-algebras and the " Fraïssé limits" of categories of C*-algebras that are sufficiently closed under tensor products. It was previously known that $\mathcal{Z}$ can be realized as the Fraïssé limit of a category of its building blocks (prime dimension-drop algebras) and unital trace-preserving embeddings.

Classification methods for homomorphism-homogeneous structures

Maja Pech

University of Novi Sad

The modern theory of homogeneous structures begins with the work of Roland Fraïssé. His fundamental results traced a road for classification of homogeneous structures, which can be witnessed by results of Gardiner, Woodrow, Lachlan, Schmerl, Cherlin, Cameron, and others. The theory developed in last 70 years is placed in the border area between combinatorics, model theory, algebra, and analysis. We turn our attention to the combinatorial pillar of this theory, namely, the work on the classification of structures for given homogeneity types. Of the special interest are homomorphism-homogeneous ones, introduced 2006 by Cameron and Nešetřil, as well as polymorphism-homogeneous structures, introduced in 2015 by Pech and Pech. In this talk, I will discuss several approaches for classifying homomorphism- and polymorphism-homogeneous structures. Special emphasis will be given to two different methods that are used for the classification of homomorphism- and polymorphism-homogeneous oriented graphs. This is joint work with Bojana Pantić and Christian Pech.